Interplay between antiferromagnetic order and spin polarization in ferromagnetic metal/electron-doped cuprate superconductor junctions

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Recently we proposed a theory of point-contact spectroscopy and argued that the splitting of zero-bias conductance peak in electron-doped cuprate superconductor point-contact spectroscopy is due to the coexistence of antiferromagnetic (AF) and *d*-wave superconducting orders [Phys. Rev. B **76**, 220504(R) (2007)]. Here we extend the theory to study the tunneling in the ferromagnetic metal/electron-doped cuprate superconductor (FM/EDSC) junctions. In addition to the AF order, the effects of spin polarization, Fermi-wave vector mismatch between the FM and EDSC regions, and effective barrier are investigated. It is shown that there exists midgap surface-state contribution to the conductance to which Andreev reflections are largely modified due to the interplay between the exchange field of ferromagnetic metal and the AF order in EDSC. Low-energy anomalous conductance enhancement can occur which could further test the existence of AF order in EDSC. Finally, we propose a more accurate formula in determining the spin-polarization value in combination with the point-contact conductance data.

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I. INTRODUCTION

Using point-contact technique to measure the spin polarization in ferromagnetic metal/conventional superconductor (FM/CS) junctions was pioneeringly done by Soulen et al.¹ and Upadhyay et al.² in 1998. Their works showed that determining the spin polarization at Fermi surface is essentially not an easy task. That leads to some definitions of spin polarization such as "tunneling polarization" proposed by Tedrow and Meservey³ and "point-contact polarization" proposed by Soulen et al.¹ One year later, Zhu and co-workers^{4,5} and Kashiwaya et al.⁶ utilized the ideas to study the spinpolarized quasiparticle transport in ferromagnet/d-wave superconductor junctions. Zhu and co-workers^{4,5} predicted that conductance resonances occur in a normal-metalferromagnet/d-wave superconductor junction and in a following paper, they further studied the junctions by solving the Bogoliubov-de Gennes (BdG) equations within an extended Hubbard model which included the proximity effect, the spin-flip interfacial scattering at the interface, and the local magnetic moment. They reported that the proximity can induce order-parameter oscillation in the ferromagnetic region. In contrast, Kashiwaya et al.⁶ focused on the spin current and spin filtering effects at the magnetic interface. In the works of Zutic and Valls,^{7,8} they first considered the effect of Fermi-wave vector mismatch (FWM) and pointed out that if one neglects FWM, the effect of spin polarization invariably leads to the suppression of Andreev reflection (AR). Among many other junction studies, Dong et al.⁹ studied a little different junction which forms a four layer sandwich, i.e., FM/I/d+is/d-wave junctions, by taking into account the roughness of the interfacial barrier and broken time-reversal symmetry states.

The pioneering works of Soulen *et al.*¹ and Upadhyay *et al.*² have inspired several experimental studies^{10–17} as well. Especially normal and ferromagnetic metal/conventional superconductors or *s*-wave superconductor (FM/*s*-wave SC)

junctions have been intensely studied experimentally and theoretical modelings [Blonder-Tinkham-Klapwijk (BTK) formula¹⁸ or its extension] had a good fitting with the conductance data. Recently Linder and Sudbø¹⁹ presented a theoretical study of FM/*s*-wave SC junction that investigated the possibility of induced triplet pairing state in the ferromagnetic metal side. They have also used the BTK approach but allowed for arbitrary magnetization strength and direction in the ferromagnet, arbitrary spin-active barrier, arbitrary FWM, and different effective masses in the two sides of the junction. As is expected, there is no retroreflection process when an exchange field is present. However, they pointed out that retroreflection can occur under some conditions.¹⁹

If one replaces the conventional superconductor by the high-temperature or *d*-wave superconductor into the junction, it will occur several novel phenomena due to its *d*-wave pairing symmetry, complex band structure, and rich magnetic properties. Of particular interest, in the electron-doped side of cuprate superconductors (EDSC), it is strongly suggested that antiferromagnetic (AF) order may coexist with the *d*-wave superconducting order, especially in the underdoped and optimally doped regimes.²⁰ In this paper, we shall explore the possible novel phenomena in the FM/EDSC junction case, taking into account the interplay between antiferromagnetic order and spin polarization. The ideas and models developed in FM/CS junctions in the literature will be applied to the current FM/EDSC junction cases.

This paper is organized as follows. In Sec. II, the basic formulation is given. We set up the condition of the junction and generalize the BdG equations to include AF order parameter. As the formal process, we utilize WKBJ approximation to obtain the more simple Andreev-type equations, which are then solved to determine the four spin-dependent reflection coefficients (detailed derivations are given in Appendix A). Formulas of charge and spin conductances are derived. Our main results and discussions are in Sec. III. In Sec. III A, the condition of midgap surface states was de-

rived (details are given in Appendix B). In Sec. III B, the effect of FWM was studied. In Secs. III C and III D, we discuss the effects of spin polarization and generalized effective barrier, respectively. It is shown that anomalous conductance enhancement can occur at low energies which could provide a further test for the existence of AF order in EDSC. In Sec. III E, a more general formula for determining the spin polarization is proposed in terms of the experimental zero-bias conductance data. Finally in Sec. IV, a brief conclusion is given.

II. FORMALISM

Our formulation is given based on the following assumptions. We consider a point contact or planar FM/I/EDSC junction where the superconductor overlayer is coated with a clean size-quantized ferromagnetic-metal overlayer of thickness *d* that is much shorter than the mean-free path *l* of normal electrons. The interface is assumed to be perfectly flat and infinitely large. Considering $l \rightarrow \infty$ limit, the discontinuity of all parameters at the interface can be neglected, except for the SC order parameter to which the proximity effect is ignored.²¹ When SC and AF orders coexist, quasiparticle (QP) excitations of an inhomogeneous superconductor can have a coupled electron-hole character associated with the coupled **k** and $\mathbf{k} + \mathbf{Q}[\mathbf{Q} = (\pi, \pi)]$ subspaces. QP states are thus governed by the generalized BdG equations^{20,22}

$$Eu_{1\sigma}(\mathbf{x}) = \hat{H}_{\sigma}u_{1\sigma}(\mathbf{x}) + \int d\mathbf{x}' \Delta(\mathbf{s}, \mathbf{r})v_{1\bar{\sigma}}(\mathbf{x}') + \Phi u_{2\sigma}(\mathbf{x}),$$

$$Ev_{1\bar{\sigma}}(\mathbf{x}) = \int d\mathbf{x}' \Delta^*(\mathbf{s}, \mathbf{r})u_{1\sigma}(\mathbf{x}') - \hat{H}_{\sigma}v_{1\bar{\sigma}}(\mathbf{x}) + \Phi v_{2\bar{\sigma}}(\mathbf{x}),$$

$$Eu_{2\sigma}(\mathbf{x}) = \Phi u_{1\sigma}(\mathbf{x}) + \hat{H}_{\sigma}u_{2\sigma}(\mathbf{x}) - \int d\mathbf{x}' \Delta(\mathbf{s}, \mathbf{r})v_{2\bar{\sigma}}(\mathbf{x}'),$$

$$Ev_{2\bar{\sigma}}(\mathbf{x}) = \Phi v_{1\bar{\sigma}}(\mathbf{x}) - \int d\mathbf{x}' \Delta^*(\mathbf{s}, \mathbf{r})u_{2\sigma}(\mathbf{x}') - \hat{H}_{\sigma}v_{2\bar{\sigma}}(\mathbf{x}),$$
(1)

where $\mathbf{s} \equiv \mathbf{x} - \mathbf{x}'$, $\mathbf{r} \equiv (\mathbf{x} + \mathbf{x}')/2$, $\hat{H}_{\sigma} \equiv -\hbar^2 \nabla^2 / 2m - E_F^{F,S} - \sigma J$, with *J* the exchange energy and $\sigma = \uparrow (\downarrow)$ for up (down) spin $(\bar{\sigma} = -\sigma)$, and Φ is the AF order parameter. $\Delta(\mathbf{s}, \mathbf{r})$ is the Cooper pair order parameter in terms of relative and centerof-mass coordinates. In the FM region, we define E_F^F $\equiv \hbar^2 q_F^2 / 2m = (\hbar^2 q_{F\uparrow}^2 / 2m + \hbar^2 q_{F\downarrow}^2 / 2m)/2$ as the spin averaged value. It differs from the value in the superconductor, E_F^S $\equiv \hbar^2 k_F^2 / 2m$, to which a FWM can occur between the FM and EDSC regions.⁸ In Eq. (1), the wave functions u_1 and v_1 are considered related to the **k** subspace, while u_2 and v_2 are related to the $\mathbf{k} + \mathbf{Q}$ subspace. Comparing to the first and second lines of Eq. (1), minus signs associated with the $\Delta(\mathbf{s}, \mathbf{r})$ term in the third and fourth lines occur due to the symmetry requirement, $\Delta(\mathbf{k} + \mathbf{Q}) = -\Delta(\mathbf{k})$, for a $d_{x^2-y^2}$ -wave superconductor in **k** space. At Fermi level, the $d_{x^2-y^2}$ -wave



FIG. 1. Schematic plot showing all possible reflection and transmission processes for an up-spin electron incident into the FM/I/ EDSC junction. An AF order is assumed to exist in the EDSC. For convenience for a *d*-wave superconductor, \mathbf{k}_x axis is chosen to be along the [110] direction. The right-bottom inset shows a given Fermi-wave vector $\mathbf{k}_F = (k_F, k_y, k_z)$ and its coupled AF wave vector $\mathbf{k}_F + \mathbf{Q} \equiv \mathbf{k}_{F\mathbf{Q}} = (-k_F, k_y, k_z)$. Both vectors are tied to the Fermi surface, which is approximated by a square (thick line). NR, AR, AF-NR, and AF-AR stand for normal reflection, Andreev reflection, antiferromagnetic-normal reflection, and antiferromagnetic-Andreev reflection, respectively. Their corresponding reflection angles are also shown. For the case of an incident down-spin electron, all spin indices just reverse.

SC gap is $\Delta(\hat{\mathbf{k}}_F) \equiv \Delta_0 \sin 2\theta$, with Δ_0 the gap magnitude and θ the azimuthal angle relative to the *x* axis.

In a *d*-wave superconductor, it is useful to consider a junction to which the superconductor surface is allied along the [110] direction. A thin insulating layer exists between the ferromagnetic metal and the superconductor (see Fig. 1) to which the barrier potential is assumed to take a delta function, $V(x)=H\delta(x)$. Considering that an up-spin electron is injected into the FM/I/EDSC junction from the ferromagnetic metal side, there are four possible reflections as follows. (a) Normal reflection (NR): reflected as electrons. (b) Andreev reflection (AR): reflected as holes due to electron and hole coupling in the k subspace. (c) Antiferromagnetic-normal reflection (AF-NR): reflected as holes due to the coupling of k and k+Q subspaces. (d) Antiferromagnetic-Andreev reflection (AF-AR): reflected as holes due to electron and hole coupling in the k+Q subspace (see Fig. 1).

In addition to the effect of AF order, AR is largely modified due to the exchange field of ferromagnetic metal when electron is not normally incident into the EDSC region. Owing to the momentum conserved parallel to the interface, Snell's law^{6,8,23} requires that

$$q_{F\sigma}\sin\theta_{N\sigma} = q_{F\bar{\sigma}}\sin\theta_{A\bar{\sigma}} = k_F\sin\theta_{S\sigma},\tag{2}$$

where $\theta_{N\sigma}$, $\theta_{A\bar{\sigma}}$, and $\theta_{S\sigma}$ are the angles of NR, AR, and transmission into the SC, respectively (see Fig. 1). Incident angle $\theta_{N\sigma}$ is typically *not* equal to the AR angle $\theta_{A\bar{\sigma}}$ except when J=0 or for normal incidence. Assuming that there is no

FWM and $q_{F\downarrow} < k_F < q_{F\uparrow}$, ranges of six normal and Andreev reflection angles are $0 < \theta_{N\uparrow} < \sin^{-1}(k_F/q_{F\uparrow}) \equiv \theta_{c2}$, $0 < \theta_{A\uparrow} < \sin^{-1}(q_{F\downarrow}/q_{F\uparrow}) \equiv \theta_{c1}$, and $0 < \theta_{S\uparrow}, \theta_{S\downarrow} < \sin^{-1}(q_{F\downarrow}/k_F)$, while $\theta_{A\downarrow}$ and $\theta_{N\downarrow}$ can be any angles. For AF reflections, the angles $\theta_{A\sigma}^{AF} = \pi - \theta_{A\sigma}$ and $\theta_{N\sigma}^{AF} = \theta_{N\sigma}$, respectively. It is noted that when $\theta_{N\uparrow}$ is within the range $\theta_{c1} < \theta_{N\uparrow} < \theta_{c2}$, *x* component of the wave vector, $\sqrt{q_{F\downarrow}^2 - k_F^2} \sin^2 \theta_{S\uparrow}$, becomes purely imaginary for the AR process.^{6,8} Although spin-down electron as a propagating wave is impossible for AR, it can still transmit into the superconductor side.

As emphasized by Kashiwaya *et al.*,⁶ one can define two types of conductance in a FM, namely, the *charge* and *spin* conductances. As a matter of fact, the normalized angle and spin-dependent tunneling charge conductance are given by

$$C_{q\sigma} = 1 - |R_{N\sigma}|^2 + a_{\sigma} |R_{A\bar{\sigma}}|^2 + |R_{N\sigma}^{AF}|^2 - a_{\sigma} |R_{A\bar{\sigma}}^{AF}|^2, \quad (3)$$

where $a_{\downarrow} \equiv 1$ and $a_{\uparrow} \equiv L_{\downarrow} \lambda_{2\downarrow} / L_{\uparrow} \lambda_{1\uparrow}$, with $\lambda_{1\uparrow} = \cos \theta_{N\uparrow} / \cos \theta_{S\uparrow}$, $\lambda_{2\downarrow} = \cos \theta_{A\downarrow} / \cos \theta_{S\uparrow}$, and $L_{\sigma} = \sqrt{(q_F/k_F)(1 - \sigma J/E_F^F)}$. Detailed derivations of all four reflection coefficients $(R_{N\sigma}, R_{A\bar{\sigma}}, R_{N\sigma}^{AF}, \text{ and } R_{A\bar{\sigma}}^{AF})$ are given in Appendix A. Similarly, the normalized angle and spin-dependent spin conductance is given by

$$C_{s\sigma} = 1 - |R_{N\sigma}|^2 - a_{\sigma}|R_{A\overline{\sigma}}|^2 + |R_{N\sigma}^{AF}|^2 - a_{\sigma}|R_{A\overline{\sigma}}^{AF}|^2.$$
(4)

Comparing to the results of charge conductance in Eq. (3), due to the spin imbalance induced by the exchange field, different signs of $R_{A\bar{\sigma}}$ terms occur in the spin conductances. Consequently normalized total charge (spin) conductance is given by

$$G_{q(s)}(E) = G_{q(s)\uparrow}(E) \pm G_{q(s)\downarrow}(E), \qquad (5)$$

where + (-) sign is for charge (spin) channel and

$$G_{q(s)\sigma}(E) = \frac{1}{G_{q(s)}^{N}} \int_{\alpha}^{\beta} d\theta_{N\sigma} \cos \theta_{N\sigma} C_{q(s)\sigma}(E, \theta_{N\sigma}) P_{\sigma}.$$
 (6)

The lower and upper integration limits of α and β are restricted by Snell's law (as discussed before) or experimental setup. In practice, integration over two separate ranges of incident angle, i.e., $0 < |\theta_{N\sigma}| < \theta_{c1}$ and $\theta_{c1} < |\theta_{N\sigma}| < \theta_{c2}$ should be carried and results are added up to the total conductance. In Eq. (6), the normal-state charge (spin) conductance

$$G_{q(s)}^{N} = \int_{-\pi/2}^{\pi/2} d\theta_{N\sigma} \cos \theta_{N\sigma} [C_{N\uparrow} P_{\uparrow} \pm C_{N\downarrow} P_{\downarrow}], \qquad (7)$$

where

$$C_{N\sigma}(\theta_{N\sigma}) = \frac{4\lambda_1 L_{\sigma}}{|1 + \lambda_1 L_{\sigma} + 2iZ|^2},$$
(8)

with $Z=mH/\hbar^2 k_F$ the barrier. In both Eqs. (6) and (7), we have presented a factor $P_{\sigma}=(E_F^F+\sigma J)/2E_F^F$, which can be interpreted as the probability of spin- σ incident electron as a function of the exchange energy.^{6,8,19} When J=0, $P_{\uparrow}=P_{\downarrow}=1/2$.

In addition to the conductances, the normalized *total* charge (spin) current can be given by

(9)

 $I_{q(s)} = I_{q(s)\uparrow} \pm I_{q(s)\downarrow},$

$$I_{q(s)\sigma} = \frac{1}{I_{q(s)}^{N}} \int_{-\infty}^{\infty} dE \int_{\alpha}^{\beta} d\theta_{N\sigma} \cos \theta_{N\sigma} C_{q(s)\sigma}(E, \theta_{N\sigma}) P_{\sigma} q_{F\sigma},$$
(10)

with

where

$$I_{q(s)}^{N} = \int_{-\infty}^{\infty} dE \int_{-\pi/2}^{\pi/2} d\theta_{N\sigma} \cos \theta_{N\sigma} [C_{N\uparrow} P_{\uparrow} q_{F\uparrow} \pm C_{N\downarrow} P_{\downarrow} q_{F\downarrow}].$$
(11)

Charge and spin currents and their conversion are important probes for spin-related phenomena such as those in spin Hall effect.

III. RESULTS AND DISCUSSIONS

Both charge and spin conductances are important probes for tunneling in spin-polarized junctions. In this paper, we will focus on the charge conductance however. Moreover, for simplicity, all the results presented are for normal incidence $(\theta_{N\sigma}=0)$.

A. Midgap surface states

Detailed derivations of the midgap surface states (MSSs) in the current FM/EDSC junction are given in Appendix B. Basically it is an extension of the works of Hu²¹ and Liu and Wu.²⁰ The boundary condition that leads to the MSS is the wave function $\psi_{N\sigma}(x=-d)=0$ for a free boundary at x=-d. Consequently one obtains the following condition for the MSS (see Appendix B):

$$e^{-2ik_{1\sigma}d}E_{+} + e^{2ik_{1\sigma}d}E_{-} = 2\Phi, \qquad (12)$$

where $E_{\pm} \equiv E \pm \varepsilon'_{\sigma}$, with $\varepsilon'_{\sigma} = \sqrt{(E + \sigma J)^2 - \Delta^2 - \Phi^2}$ and incident spin- σ electron is assumed to have wave vector $k_{1\sigma}$ along the *x* direction.

In case of J=0, the result is reduced to our previous case without spin polarization.²⁰ In case of $J=\Phi=0$, the result is reduced to the case of Hu,²¹ i.e.,

$$e^{4ik_1d} = -\frac{E+\varepsilon'}{E-\varepsilon'},\tag{13}$$

where $\varepsilon' \equiv \sqrt{E^2 - \Delta^2}$. The most crucial result of the above is that there exists a zero-energy state which is responsible for the zero-bias conductance peak (ZBCP) widely observed in hole-doped $d_{x^2-y^2}$ -wave cuprate superconductors.²¹ When J=0 but $\Phi \neq 0$, zero-energy state no longer exists such that the energy of the existing state is always finite ($E=\Phi$ in the limit of d=0). This leads to the splitting of the ZBCP. When J is also finite, there will be further effect caused by spin polarization although the splitting peak remains at $E=\Phi$ in the limit of d=0. It is interesting to note that beyond the quasiclassical approximation, a more accurate calculation for the surface bound-state energies in $d_{x^2-y^2}$ -wave and other uncon-



FIG. 2. (Color online) Effect of FWM on normalized charge conductance spectra G_q for various wave-vector mismatch values L_0 with fixed barrier Z=0, AF order Φ =0.5 Δ_0 , and spin polarization X=0.5.

ventional cuprate superconductors was reported by Walker *et al.*²⁴

B. Effect of Fermi-wave-vector mismatch

Tunneling conductances are in general strongly modified by the effect of FWM.^{8,19} In our case, due to the presence of the AF order, the conductance spectra are somewhat different from those obtained by Zutic and Valls⁸ and Linder and Sudbo.¹⁹ Here we introduce a parameter

$$L_0 \equiv \frac{q_F}{k_F} \tag{14}$$

to account for the effect of FWM. Both L_0 , greater and smaller than 1, cases are considered. As shown in Figs. 2–4 for the normalized charge conductance G_q , the effect of FWM is typically strong when $L_0 < 1$, while it has relatively minor effect when $L_0 > 1$ (that is, G_q changes little from the no FWM $L_0=1$ case). As first pointed out by Blonder and Tinkham,²⁵ FWM can be interpreted as a type of barrier which could enhance the conductance near zero bias.

In our previous paper,²⁰ it was shown that ZBCP of a $d_{x^2-y^2}$ -wave superconductor can be split by the AF order Φ . No spin-active barrier,^{6,19} external magnetic field, and spinpolarization effects were considered in our previous case though. Previously Zutic and Valls⁸ had given a detailed analysis of the FWM effect on the conductance in ferromagnet/s-wave and d-wave superconductor junctions. Here we show how FWM influences the conductance in the current case and point out the key physics. Figure 2 plots G_q for various L_0 with barrier Z=0, AF order $\Phi = 0.5\Delta_0$, and spin polarization X=0.5 [see Eq. (17) for the definition of X]. One sees that the effect of FWM is most noticeable at large FWM ($L_0=0.2$ case) to which a ZBCP is developed, while the spectra are humdrum when $L_0 \ge 1$. Since no barrier (Z =0) is considered, no effect of AF order and spin polarization is seen in terms of peak splitting. Note that normalized



FIG. 3. (Color online) Effect of FWM on normalized charge conductance spectra G_q for various wave-vector mismatch values L_0 with fixed barrier Z=1, AF order Φ =0, and spin polarization X=0. This can be considered as the case of hole-doped cuprate superconductors without AF order and in the absence of spin polarization.

zero-bias conductance is not equal to 2 due to the presence of AF order and spin polarization. In order to compare to the case of hole-doped high- T_c superconductors (without AF order), Fig. 3 plots G_q at different values of L_0 with $\Phi=X=0$ and Z=1. One sees that ZBCP is largely enhanced by the FWM effect (see $L_0=0.2$ case). Thus FWM can significantly enhance the number of midgap surface states near zero-bias voltage.

Aiming to electron-doped cuprate superconductors, Fig. 4 shows the effect of FWM on the splitting peak when the AF order is present ($\Phi=0.5\Delta_0$). Here barrier Z=1 and spin polarization X=0.5. In contrast to the case of $\Phi=0$ in Fig. 3, FWM actually reduces the number of midgap surface states



FIG. 4. (Color online) Effect of FWM on normalized charge conductance spectra G_q for various wave-vector mismatch values L_0 with fixed barrier Z=1, AF order $\Phi=0.5\Delta_0$, and spin polarization X=0.5. FWM causes the reduction of conductance at zero bias, while enhancing the splitting peak associated with the AF order.

near zero bias. At the same time, it enhances the strength of the splitting peak associated with the AF order. Following the idea of Blonder and Tinkham²⁵ such that FWM can be interpreted as a type of barrier, the enhancement of ZBCP in Fig. 3 and the reduction of zero-bias conductance in Fig. 4 are a natural outcome at large FWM. In principle, the effect of FWM should be included when a serious calculation is performed for spin-polarized conductances.

C. Effect of spin polarization

In the literature, there exist different definitions of spin polarization. One example is the "tunneling polarization" proposed by Tedrow and Meservey.³ In point-contact experiment, the more suitable definition is the so-called "contact polarization"¹

$$P_{c} = \frac{N_{\uparrow}(E_{F})v_{F\uparrow} - N_{\downarrow}(E_{F})v_{F\downarrow}}{N_{\uparrow}(E_{F})v_{F\uparrow} + N_{\downarrow}(E_{F})v_{F\downarrow}},$$
(15)

where $v_{F\sigma}$ and $N_{\sigma}(E_F)$ are, respectively, the Fermi velocity and DOS at Fermi level for spin- σ electron. Since $I_{\sigma} \propto N_{\sigma}(E_F)v_{F\sigma}$, Eq. (15) is identical to

$$P_c = \frac{I_{\uparrow} - I_{\downarrow}}{I_{\uparrow} + I_{\downarrow}}.$$
 (16)

However, the most natural definition of spin polarization is

$$X \equiv \frac{N_{\uparrow}(E_F) - N_{\downarrow}(E_F)}{N_{\uparrow}(E_F) + N_{\downarrow}(E_F)}.$$
(17)

In ballistic point-contact situation, the electron density of states in the presence of an exchange field can be written as $N_{\sigma}(E_F) = q_{F\sigma}^2 A/4\pi$, where A is the area of the interface. Thus $X = J/E_F^F$, with $E_F^F \equiv \hbar^2 q_F^2/2m = (\hbar^2 q_{F\uparrow}^2/2m + \hbar^2 q_{F\downarrow}^2/2m)/2$.¹⁷ In Sec. III E, we will show that spin polarization X can be determined by a general formula in combination with the experimental conductance data.

Note that current quasiparticle wave function of BdG equations has four components due to the presence of the AF order. In the limit of Z=0 and without spin polarization (X =0), normalized charge conductance has a value of 2 as expected (see Fig. 5). With a finite AF order (Φ =0.5 Δ_0), the resulting effective gap magnitude is about $\tilde{\Delta} \approx 1.12\Delta_0$ (see Fig. 5). In general, at $E < \tilde{\Delta}$, effect of spin polarization is to suppress the conductance. When FWM is absent (L_0 =1) together with Z=0, normal reflection has no contribution and Andreev reflection actually dominates the tunneling process for $E < \tilde{\Delta}$.¹⁸ In our current case, Andreev reflection involves contributions from both R_A and R_A^{AF} channels.

The most interesting results occur when the barrier Z is finite. When the AF order $\Phi=0$ (as for the case of holedoped cuprate superconductors) to which $R_N^{AF} = R_A^{AF} = 0$, ZBCP appears whose (normalized) strength is largely suppressed due to the strong spin-polarization effect (see Fig. 6). However, as seen in Fig. 7, when AF order is finite (Φ =0.5 Δ_0), in contrast, the strengths of both the zero-bias conductance and the splitting peak turn out to get enhanced by the strong spin-polarization effect. This "anomalous conduc-



FIG. 5. (Color online) Effect of spin polarization on normalized charge conductance spectra G_q for various spin-polarization values X with fixed barrier Z=0, AF order Φ =0.5 Δ_0 , and without FWM (L_0 =1).

tance enhancement" phenomenon is in drastic contrast as compared to the ZBCP associated with $\Phi=0$ case (Fig. 6). These somewhat surprising results arise due to a significant increase of $|R_N^{AF}|$ and at the same time, a significant decrease of $|R_A^{AF}|$ for large X cases—a consequence of the interplay between AF order and spin polarization. Since $|R_N^{AF}|$ contributes positively to the conductance, while $|R_A^{AF}|$ contributes negatively to the conductance [see Eq. (3)], resultantly they cause the anomalous conductance enhancement at low energies ($E \le \Phi$). It should be emphasized that this low-energy conductance enhancement is not due to the spin-flip effect which is not considered in this paper. At higher energies, $E > \Phi$, the conductances behave more normally such that they get suppressed due to the spin-polarization effect. Anomalous conductance enhancement at low energies can serve as a



FIG. 6. (Color online) Effect of spin polarization on normalized charge conductance spectra G_q for various spin-polarization values X with fixed barrier Z=1, AF order Φ =0, and without FWM (L_0 = 1). This is considered an example of the hole-doped cuprate superconductor without AF order and FWM.



FIG. 7. (Color online) Effect of spin polarization on normalized charge conductance spectra G_q for various spin-polarization values X with fixed barrier Z=1, AF order Φ =0.5 Δ_0 , and without FWM (L_0 =1). Low-energy anomalous conductance enhancement arises due to AF contributions (see text for details).

test to see whether there is an significant AF order in electron-doped cuprate superconductors.

Interface barrier and band structure are in general having strong effect on spin polarization. Kant *et al.*¹² have built an "extended interface" model to illustrate the decay of spin polarization. Besides, Mazin had a detailed discussion on the definition of spin polarization and band-structure effects in spin polarization.²⁶

D. Effect of effective barrier

In the study of the tunneling transition in Cu-Nb point contacts, Blonder and Tinkham²⁵ pointed out that barrier is not the only source for normal reflection and in a more realistic system, one should consider "impedance" or FWM as well which results in normal reflection even with no barrier present. They proposed an effective barrier $Z_{\rm eff} = [Z^2 + (1 - r)^2/4r]^{1/2}$, where *r* is the Fermi velocity ratio. They showed that effective barrier has an obvious effect on the conductance when $E < \Delta_0$, as shown in Fig. 2 of Ref. 25. Here we generalize their idea to consider a spin, FWM, and angle dependent effective barrier $Z_{\rm eff}^{8,25}$

$$Z_{\rm eff} \equiv [Z^2 + (1 - L_{\sigma})^2 / 4L_{\sigma}]^{1/2} / \cos \theta_{S\sigma}, \qquad (18)$$

where $L_{\sigma} = q_{F\sigma}/k_F$ corresponds to the spin-dependent FWM. It is noted that we are not considering the spin-active barrier which has spin filtering effects and can lead to the ZBCP splitting.^{6,19} Instead we propose a possible alternative mechanism to account for the decay of spin polarization. Based on the generalized effective barrier, spin-up and spin-down particles experience different strengths of effective barrier that causes spin-up and spin-down currents to decrease at different speed as compared to the current in the absence of barrier. Consequently, $Z_{\rm eff}$ can modify the values of $I_{\uparrow} - I_{\downarrow}$ (and thus P_c) dramatically. With this strong effect at work, the decay of spin polarization should not be dominant



FIG. 8. (Color online) Effect of effective barrier $Z_{\rm eff}$ on normalized charge conductance spectra G_q for various values of FWM L_0 . AF order $\Phi = 0.5\Delta_0$ and spin polarization X = 0.5. The bare barrier Z is set to zero, while $Z_{\rm eff}$ is given by Eq. (18).

by the spin-flitting process in the point-contact spin-polarization case.

As seen in Eq. (18), $Z_{\rm eff}$ can differ significantly from Z, especially when Z is small. Essentially their difference can be measured by spin-polarized tunneling experiments. In Fig. 8, we compare the effects of Z and $Z_{\rm eff}$ on the conductance to bare barrier Z set to zero and vary the FWM L_0 value. For Z=0 and $\theta_{S\sigma}=0$, $Z_{\rm eff}=[(1-L_{\sigma})^2/4L_{\sigma}]^{1/2}$ [see Eq. (18)]. In our case, we have also included AF order and spin polarization. The difference is most noticeable when FWM is large $(L_0=0.2 \text{ case})$. Since Z=0, AF order and spin polarization have little effect at small FWM. However, when FWM is large, AF order and spin polarization can have a strong effect such that a splitting peak can develop at $E \approx \Phi = 0.5\Delta_0$ with the effective barrier $Z_{\rm eff}$ (see Fig. 8). This supports the idea of Blonder and Tinkham²⁵ of "impedance" mismatch which enhances the normal reflection.

E. General formula for determining the spin polarization

Based on the phenomenon of Andreev reflection, Soulen *et al.*¹ proposed a formula for determining the point-contact spin polarization P_c [see Eqs. (15) and (16)] when the normalized zero-bias conductance data is compared. Their original form was

$$G(0)/G_N = 2(1 - P_c), \tag{19}$$

which is valid only when FWM is absent.¹¹ Since Andreev reflection could be strongly modified due to the FWM effect, it is useful to replace Eq. (19) by

$$G(0)/G_N = [1 + |R_A|^2 - |R_A^{AF}|^2](1 - P_c), \qquad (20)$$

where R_A and R_A^{AF} are the AR and AF-AR coefficients, respectively. Equation (20) can be reduced to Eq. (19) when the exchange energy J is set to zero in R_A and the AF order Φ is set to zero in R_A^{AF} . Note also that the parameter X should be set to zero when the "contact polarization" P_c is determined under the idea of Soulen *et al.*¹

Here we propose a more general formula for determining the spin polarization

$$G(0)/G_N = A_{\uparrow} + A_{\perp}, \qquad (21)$$

where

$$A_{\uparrow} = \int_{\alpha}^{\beta} d\theta_{N\sigma} \cos \theta_{N\sigma} (1 + a_{\uparrow} | R_{A\downarrow} |^2 - a_{\uparrow} | R_{A\downarrow}^{AF} |^2) P_{\uparrow} \quad (22)$$

and

$$A_{\downarrow} = \int_{\alpha}^{\beta} d\theta_{N\sigma} \cos \theta_{N\sigma} (1 + |R_{A\uparrow}|^2 - |R_{A\uparrow}^{AF}|^2) P_{\downarrow}.$$
 (23)

Here $R_{A\sigma} = R_{A\sigma}(L_0, X, \Phi, \theta_{N\sigma})$ and $R_{A\sigma}^{AF} = R_{A\sigma}^{AF}(L_0, X, \Phi, \theta_{N\sigma})$ with E=0. Equation (21) is a natural result of our earlier formalism. It is regarded as the generalization of Eq. (19) of Soulen *et al.*,¹ which includes the effects of FWM, spin polarization, AF order, as well as the incident angle.

IV. CONCLUSIONS

Tunneling experiment provides a useful tool for probing the properties of a superconductor such as the magnitude and symmetry of the superconducting order parameter, quasiparticle density of states, and any existing competing orders. In fact, tunneling experiment is also a powerful probe for investigating the spin-charge separation in connection with the spin-injection techniques. This involves both charge imbalance and spin imbalance studies.

In this paper, we have presented a detailed study of the tunneling conductance spectra of ferromagnetic metal/ electron-doped superconductor junctions, taking into account an AF order existing in the electron-doped superconductor. Interesting result, such as low-energy anomalous conductance enhancement, occurs as a result of the interplay between AF order and spin polarization (see Fig. 7). These results in turn provide a further opportunity to test whether there is an significant AF order in electron-doped cuprate superconductors.

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APPENDIX A: REFLECTION COEFFICIENTS

Under the WKBJ approximation, $^{18,21,27-31}$ the wave functions in the generalized BdG [Eq. (1)] can be approximated by

$$\begin{pmatrix} u_{1\sigma} \\ v_{1\bar{\sigma}} \\ u_{2\sigma} \\ v_{2\bar{\sigma}} \end{pmatrix} = \begin{pmatrix} e^{i\mathbf{k}_{F}\cdot\mathbf{r}}\widetilde{u}_{1\sigma} \\ e^{i\mathbf{k}_{F}\cdot\mathbf{r}}\widetilde{v}_{1\bar{\sigma}} \\ e^{i\mathbf{k}_{FQ}\cdot\mathbf{r}}\widetilde{u}_{2\sigma} \\ e^{i\mathbf{k}_{FQ}\cdot\mathbf{r}}\widetilde{v}_{2\bar{\sigma}} \end{pmatrix}.$$
 (A1)

Thus one obtains a set of Andreev equations in the x direction,

$$\begin{split} & E \widetilde{u}_{1\sigma}(x) = H_{\sigma} \widetilde{u}_{1\sigma}(x) + \Delta(\hat{\mathbf{k}}_F) \widetilde{v}_{1\overline{\sigma}}(x) + \Phi \widetilde{u}_{2\sigma}(x), \\ & E \widetilde{v}_{1\overline{\sigma}}(x) = \Delta^*(\hat{\mathbf{k}}_F) \widetilde{u}_{1\sigma}(x) - H_{\sigma} \widetilde{v}_{1\overline{\sigma}}(x) + \Phi \widetilde{v}_{2\overline{\sigma}}(x), \end{split}$$

$$E\tilde{u}_{2\sigma}(x) = \Phi\tilde{u}_{1\sigma}(x) - H_{\sigma}\tilde{u}_{2\sigma}(x) + \Delta(\hat{\mathbf{k}}_{F\mathbf{0}})\tilde{v}_{2\bar{\sigma}}(x),$$

$$E\tilde{v}_{2\bar{\sigma}}(x) = \Phi\tilde{v}_{1\bar{\sigma}}(x) + \Delta^*(\hat{\mathbf{k}}_{F\mathbf{Q}})\tilde{u}_{2\sigma}(x) + H_{\sigma}\tilde{v}_{2\bar{\sigma}}(x), \quad (A2)$$

where $H_{\sigma} = -\frac{i\hbar^2 k_F}{m} \frac{d}{dx} - \sigma J$ and *x* is the coordinate normal to the interface. The $d_{x^2-y^2}$ -wave SC gap $\Delta(\hat{\mathbf{k}}_F) = -\Delta(\hat{\mathbf{k}}_{FQ}) \equiv \Delta_0 \sin 2\theta$ with θ the azimuthal angle relative to the *x* axis. In obtaining Eq. (A2), the Fourier transform of the Cooper pair order parameter $\Delta(\mathbf{s}, \mathbf{r})$ from relative coordinate **s** to **k** space is assumed to take the form $\Delta(\mathbf{k}, \mathbf{r}) = \Delta(\hat{\mathbf{k}}_F)\Theta(x)$, with $\Theta(x)$ the Heaviside step function.^{21,27}

Solving Eq. (A2), one obtains four eigenvectors which build up the spin- σ wave function in the superconductor region (x>0),⁸

$$\psi_{S\sigma}(x) = \begin{bmatrix} c_{1\sigma} \begin{pmatrix} \Delta \\ E_{-} \\ 0 \\ \Phi \end{pmatrix} + c_{2\sigma} \begin{pmatrix} E_{+} \\ \Delta \\ \Phi \\ 0 \end{pmatrix} \end{bmatrix} e^{ik^{+}x} + \begin{bmatrix} c_{3\sigma} \begin{pmatrix} E_{-} \\ -\Delta \\ \Phi \\ 0 \end{pmatrix} + c_{4\sigma} \begin{pmatrix} -\Delta \\ E_{+} \\ 0 \\ \Phi \end{pmatrix} \end{bmatrix} e^{-ik^{-}x}.$$
 (A3)

Here $E_{\pm} \equiv E \pm \varepsilon_{\sigma}$, with $\varepsilon_{\sigma} = \sqrt{E^2 - \Delta^2 - \Phi^2}$, $\Delta \equiv \Delta(\hat{\mathbf{k}}_F)$, $k^+ = k^- = k_F \cos \theta_{S\sigma}$, and $c_{i\sigma}$ are coefficients of the corresponding waves. As pointed out by Blonder *et al.*,¹⁸ there is no need to normalize the coefficients as it just complicates the calculation. If we set $\Phi = J = 0$ and normalize the coefficients, it will reduce to the case for a typical N/I/S junction.^{18,27,31}

Since we consider that there is an AF order in the EDSC side, an incident electron from the FM side will have four possible reflections.²⁰ The spin- σ wave function in the FM side (x < 0) with incident angle $\theta_{N\sigma}$ can thus be written as^{6,18,27}

$$\Psi_{N\sigma}(x) = \begin{pmatrix} e^{iq_{F\sigma}\cos\theta_{N\sigma}x} + R_{N\sigma}e^{-iq_{F\sigma}\cos\theta_{N\sigma}x} \\ R_{A\bar{\sigma}}e^{iq_{F\bar{\sigma}}\cos\theta_{A\bar{\sigma}}x} \\ R_{N\sigma}^{A\bar{F}}e^{iq_{F\sigma}\cos\theta_{N\sigma}x} \\ R_{A\bar{\sigma}}^{A\bar{F}}e^{-iq_{F\bar{\sigma}}\cos\theta_{A\bar{\sigma}}x} \end{pmatrix}, \quad (A4)$$

where $R_{N\sigma}$, $R_{A\bar{\sigma}}$, $R_{N\sigma}^{AF}$ and $R_{A\bar{\sigma}}^{AF}$ are amplitudes of NR, AR, AF-NR, and AF-AR, respectively. Applying the following boundary conditions:

$$\psi_{N\sigma}(x)|_{x=0^-} = \psi_{S\sigma}(x)|_{x=0^+}$$

$$\frac{2mH}{\hbar^2}\psi_{S\sigma}(x)\big|_{x=0^+} = \left.\frac{d\psi_{S\sigma}(x)}{dx}\right|_{x=0^+} - \left.\frac{d\psi_{N\sigma}(x)}{dx}\right|_{x=0^-},$$
(A5)

the four reflection amplitudes (coefficients) are solved to be

$$\begin{split} R_{N\sigma} &= \frac{E_{-}(1-L_{\sigma}\lambda_{1\sigma}+2iZ_{\theta})B}{(1+L_{\sigma}\lambda_{1\sigma}+2iZ_{\theta})D} - \frac{\Delta(1+L_{\bar{\sigma}}\lambda_{2\bar{\sigma}}+2iZ_{\theta})A}{(1+L_{\sigma}\lambda_{1\sigma}+2iZ_{\theta})D} \\ &- \frac{1-L_{\sigma}\lambda_{1\sigma}-2iZ_{\theta}}{1+L_{\sigma}\lambda_{1\sigma}+2iZ_{\theta}}, \\ R_{A\bar{\sigma}} &= \frac{\Delta(1+L_{\sigma}\lambda_{1\sigma}-2iZ_{\theta})B}{(1+L_{\bar{\sigma}}\lambda_{2\bar{\sigma}}-2iZ_{\theta})D} + \frac{E_{-}(1-L_{\bar{\sigma}}\lambda_{2\bar{\sigma}}-2iZ_{\theta})A}{(1+L_{\bar{\sigma}}\lambda_{2\bar{\sigma}}-2iZ_{\theta})D}, \end{split}$$

$$R_{N\sigma}^{\rm AF} = \frac{\Phi B}{D},$$

$$R_{A\overline{\sigma}}^{\rm AF} = \frac{\Phi A}{D},$$
(A6)

where

$$A = 2\Delta L_{\sigma}\lambda_{1\sigma} [1 - L_{\sigma}L_{\bar{\sigma}}\lambda_{1\sigma}\lambda_{2\bar{\sigma}} + 4Z_{\theta}^{2} + 2iZ_{\theta}(L_{\sigma}\lambda_{1\sigma} + L_{\bar{\sigma}}\lambda_{2\bar{\sigma}})],$$

$$B = 2L_{\sigma}\lambda_{1\sigma} [2L_{\bar{\sigma}}\lambda_{2\bar{\sigma}}E + \varepsilon(1 + L_{\bar{\sigma}}^{2}\lambda_{2\bar{\sigma}}^{2})],$$

$$D = \Delta^{2} [(1 - L_{\sigma}L_{\bar{\sigma}}\lambda_{1\sigma}\lambda_{2\bar{\sigma}} + 4Z_{\theta}^{2})^{2} + 4Z_{\theta}^{2}(L_{\sigma}\lambda_{1\sigma} + L_{\bar{\sigma}}\lambda_{2\bar{\sigma}})^{2}] + [2L_{\sigma}\lambda_{1\sigma}E + 4\varepsilon Z_{\theta}^{2} + \varepsilon(1 + L_{\sigma}^{2}\lambda_{1\sigma}^{2})] \times [2L_{\bar{\sigma}}\lambda_{2\bar{\sigma}}E + 4\varepsilon Z_{\theta}^{2} + \varepsilon(1 + L_{\sigma}^{2}\lambda_{2\bar{\sigma}}^{2})].$$
(A7)

Moreover $Z_{\theta} = Z/\cos \theta_{S\sigma}$ with the barrier $Z = mH/\hbar^2 k_F$, $\lambda_{1\sigma} = \cos \frac{\theta_{N\sigma}}{\cos \theta_{S\sigma}}$, $\lambda_{2\bar{\sigma}} = \cos \frac{\theta_{A\bar{\sigma}}}{\cos \theta_{S\sigma}}$, and $L_{\sigma} = \sqrt{q_F/k_F - \sigma(q_F/k_F)}(J/E_{FN})$. It is interesting to note in Eq. (A6) that $R_{N\sigma}^{AF}$ and $R_{A\bar{\sigma}}^{AF}$ are proportional to the AF order Φ as is expected.

APPENDIX B: MIDGAP SURFACE STATES

Following Ref. 21, we first assume that

$$\begin{pmatrix} \tilde{u}_{l\sigma} \\ \tilde{\upsilon}_{l\sigma} \end{pmatrix} = e^{-\gamma_{\sigma} x} \begin{pmatrix} \hat{u}_{l\sigma} \\ \hat{\upsilon}_{l\sigma} \end{pmatrix}, \tag{B1}$$

where γ_{σ} is the attenuation constant for $|E(\mathbf{q}_{F\sigma})| < \sqrt{|\Delta(\hat{\mathbf{k}}_{F})|^2 + \Phi^2}$. With Eq. (B1), Eq. (A2) becomes

$$E\begin{pmatrix} \hat{u}_{1\sigma} \\ \hat{v}_{1\bar{\sigma}} \\ \hat{u}_{2\sigma} \\ \hat{v}_{2\bar{\sigma}} \end{pmatrix} = \begin{pmatrix} h & \Delta & \Phi & 0 \\ \Delta & -h & 0 & \Phi \\ \Phi & 0 & -h & -\Delta \\ 0 & \Phi & -\Delta & h \end{pmatrix} \begin{pmatrix} \hat{u}_{1\sigma} \\ \hat{v}_{1\bar{\sigma}} \\ \hat{u}_{2\sigma} \\ \hat{v}_{2\bar{\sigma}} \end{pmatrix}$$
(B2)

for the superconducting overlayer (x>0). Here $h=\varepsilon'_{\sigma}-\sigma J$ with $\varepsilon'_{\sigma}=i\hbar^2m^{-1}\gamma_{\sigma}q_F\cos\theta_{N\sigma}$. The wave-vector components parallel to the interface are conserved for all possible processes.

Solving Eq. (B2), one obtains double degenerate eigenvalues $E = \pm \sqrt{\Delta^2 + \Phi^2 + \varepsilon_{\sigma}'^2} - \sigma J$, where + (-) corresponds to the electron (hole)-like QP excitation. Similar to the wave function (A3), superposition of the four eigenstates makes up the formal wave function for the superconductor overlayer (x > 0),

$$\psi_{S\sigma}(x) = \begin{bmatrix} c_{1\sigma} \begin{pmatrix} \Delta \\ E_{-} \\ 0 \\ \Phi \end{pmatrix} + c_{2\sigma} \begin{pmatrix} E_{+} \\ \Delta \\ \Phi \\ 0 \end{pmatrix} \end{bmatrix} e^{-\gamma_{\sigma}x} e^{ik^{+}x} + \begin{bmatrix} c_{3\sigma} \begin{pmatrix} E_{-} \\ -\Delta \\ \Phi \\ 0 \end{pmatrix} + c_{4\sigma} \begin{pmatrix} -\Delta \\ E_{+} \\ 0 \\ \Phi \end{pmatrix} \end{bmatrix} e^{-\gamma_{\sigma}x} e^{-ik^{-}x}.$$
(B3)

Here $E_{\pm} \equiv E \pm \varepsilon'_{\sigma}$, with $\varepsilon'_{\sigma} = \sqrt{(E + \sigma J)^2 - \Delta^2 - \Phi^2}$, c_i are coefficients of the corresponding waves, and $k^+ = k^- = k_F \cos \theta_{S\sigma}$. At the interface, the wave functions of FM and superconductor meet ideal continuity $\psi_{N\sigma}(x=0) = \psi_{S\sigma}(x=0)$. After some algebra, the formal wave function for the FM overlayer is obtained to be (x < 0),

$$\psi_{N\sigma}(x) = \begin{bmatrix} c_{1\sigma} \begin{pmatrix} e^{ik_{1\sigma}x}\Delta \\ e^{-ik_{1\sigma}x}E_{-} \\ 0 \\ e^{ik_{1\sigma}x}\Phi \end{pmatrix} + c_{2\sigma} \begin{pmatrix} e^{ik_{1\sigma}x}E_{+} \\ e^{-ik_{1\sigma}x}\Delta \\ e^{-ik_{1\sigma}x}\Phi \\ 0 \end{pmatrix} \end{bmatrix} e^{ik^{+}x} + \begin{bmatrix} c_{3\sigma} \begin{pmatrix} e^{-ik_{1\sigma}x}E_{-} \\ -e^{ik_{1\sigma}x}\Delta \\ e^{ik_{1\sigma}x}\Phi \\ 0 \end{pmatrix} + c_{4\sigma} \begin{pmatrix} -e^{-ik_{1\sigma}x}\Delta \\ e^{ik_{1\sigma}x}E_{+} \\ 0 \\ e^{-ik_{1\sigma}x}\Phi \end{pmatrix} \end{bmatrix} e^{-ik^{-}x},$$
(B4)

where it is assumed that incident spin- σ electron has the wave vector $k_{1\sigma}$ along the *x* direction. Considering the free boundary at x=-d, $\psi_{N\sigma}(x=-d)=0$, one thus obtains the condition for the surface bound states

$$e^{-2ik_{1\sigma}d}E_{+} + e^{2ik_{1\sigma}d}E_{-} = 2\Phi.$$
 (B5)

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